EXERCISE – V HINTS & SOLUTIONS

Sol.1 **(a)** $w = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

$4 + 5\omega^{334} + 3\omega^{365}$

$= 4 + 5\omega + 3\omega^2$

$= 1 + 3 + 3\omega + 3\omega^2 + 2\omega$

$= 1 + 2\omega$

$= 1 + (-1 + i\sqrt{3}) = i\sqrt{3}$

(b) $|z|^2 = z\bar{z}$

$|z|^2\,\omega - |\omega|^2\,z = z - \omega$

$\Rightarrow z(1 + |\omega|^2) = \omega(1 + |z|^2)$

$\therefore \frac{z}{\omega} = \frac{1+|z|^2}{1+|\omega|^2} = \text{Real} \Rightarrow \frac{z}{\omega} = \left(\frac{\bar{z}}{\omega}\right) = \frac{\bar{z}}{\omega}$

$z\bar{\omega} = \bar{z}\omega$ ...(1)

$z(\bar{z}\omega - 1) - w(\bar{\omega}z - 1) = 0$

$(z\bar{\omega} - 1)(z - \omega) = 0$ by Equation (1)

$z = \omega$ or $z\bar{\omega} = 1$

Sol.2 **(i)** $\alpha = e^{\frac{i2\pi}{7}} \Rightarrow \alpha^7 = e^{i2\pi} = 1,\ \alpha \neq 1$

or $\alpha^7 - 1 = 0$ or $(\alpha - 1)(\alpha^6 + \alpha^5 + ... + 1) = 0$

$\Rightarrow 1 + \alpha + \alpha + ... + \alpha^6 = 0$

Also $1 + \alpha^k + \alpha^{2k} + ... + \alpha^{6k}$

$= \frac{1-(\alpha^k)^7}{1-\alpha^k} = \frac{1-\alpha^{7k}}{1-\alpha^k} = \frac{1-1}{1-\alpha^k} = 0$ where $k \neq 7m$

$f(x) = A_0 + \sum_{k=1}^{20} A_k x^k$

Replace x by x, αx, a^2x ... $\alpha^6 x$ is the above and add then

L.H.S. $= (A_0 + A_0 + ...) + \sum_{k=1}^{20} A_x x^k (1 + \alpha^k + \alpha^{2k} ... + \alpha^{6k})$

$= 7A_0 + 0$

$= 7A_0$

Which is indepoendent of α

(ii) Let roots $\alpha + i\beta,\ \alpha - i\beta,\ \gamma$

$\alpha + i\beta + \alpha - i\beta + \gamma = 0$

$2\alpha + \gamma = 0$

$\gamma = -2\alpha$

again $(-2\alpha)^3 + q(-2\alpha) + r = 0$

$(2\alpha)^3 + q(2\alpha) - r = 0$

$x^3 + qx - r = 0$

Sol.3 **(a)** $|z_1| = |z_2| = |z_3| = 1$

$\bar{z}_1 = \frac{1}{z_1}$; $\bar{z}_2 = \frac{1}{z_2}$; $\bar{z}_3 = \frac{1}{z_3}$

$\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$

$|\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 1$

$|\overline{\bar{z}_1 + \bar{z}_2 + \bar{z}_3}| = 1 \quad \Rightarrow |z_1 + z_2 + z_3| = 1$

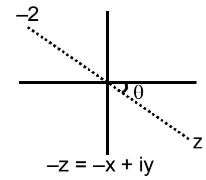
(b) arg (z) < 0

$\Rightarrow z = x - iy$

arg (–z) – arg (z)

$\pi - \theta - (-\theta)$

$= \pi + \theta + \theta = \pi$



Sol.4 $z = \cos\frac{2\pi}{2n+1} + i\sin\frac{2\pi}{2n+1}$

Clearly $z^{2n+1} = \cos 2\pi + i\sin 2\pi = 1$ ....(i)

$\alpha = \frac{z[1-(z^2)^n]}{1-z^2} = \frac{z-1}{1-z^2} = \frac{-1}{z+1}$

$\beta = \frac{z^2[1-(z^2)^n]}{1-z^2} = \frac{z-1}{1-z^2} = -\frac{-1}{z+1}$

$\beta = \frac{z^2[1-(z^2)^n]}{1-z^2} = \frac{z(z-1)}{1-z^2} = -\frac{z}{1+z}$

$\alpha + \beta = -\left(\frac{z+1}{z+1}\right) = -1 = 5$

$\alpha\beta = \frac{z}{(z+1)^2} = \frac{1}{z + \frac{1}{z} + 2}$

$z = \cos\theta + i\sin\theta$ Where $\theta = \frac{2\pi}{2n+1}$ ...(2)

and $\frac{1}{z} = \cos\theta - i\sin\theta \quad \therefore z + \frac{1}{z} = 2\cos\theta$

$\alpha\beta = \frac{1}{2\cos\theta + 2} = \frac{1}{4\cos^2\frac{\theta}{2}} = P$

Equation will be $z^2 + z + \frac{1}{4\cos^2\frac{2\pi}{2n+1}} = 0$

Sol.5 $z^{12} - 56 z^6 - 512 = 0$

Let $z^6 = t$

$t^2 - 56 t - 512 = 0$

$(t - 64)(t + 8) = 0$

$z = (64)^{1/8}, (-8)^{1/6}$

$z = 2(1)^{1/6}, \sqrt{2}(-1)^{1/6}$

$z = 2[\cos 0 + i \sin 0]^{1/6}$

$= 2\left[\cos\frac{2m\pi}{6} + i\sin\frac{2m\pi}{6}\right]$

Where m = 0, 1, 2, 3, 4, 5

Img. part corresponding to m = 4, 5 will be – ve.

Hence we choose m = 0, 1,2, 3 only.

Again $z = \sqrt{2}(-1)^{1/6} = \sqrt{2}[\cos\pi + i\sin\pi]^{1/6}$

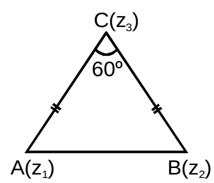
$$= \sqrt{2}\left[\cos\frac{(2m+1)}{6}\pi + i\sin\frac{(2m+1)\pi}{6}\right]$$

Ing. part corresponding to m = 3, 4, 5 will be –ve.

Hence we choose m = 0, 1, 2 only.

Sol.6 **(a)** Taking mod of both side of given eqaution

$\frac{|z_1 - z_3|}{|z_2 - z_3|} = \frac{1}{4} + \frac{3}{4} = 1$



$\therefore$ AC = BC

Hence Δ is siosceles

Also $\frac{z_1 - z_3}{z_2 - z_3} = \cos\frac{\pi}{3} - i\sin\frac{\pi}{3} = e^{-\frac{\pi}{3}i}$

or $z_2 - z_3 = (z_1 - z_3)\, e^{\frac{\pi}{3}i}$

Anticlockwise rotation implies $\angle ABD = 60°$

Hence isosceles O is equilateral.

(b) $z = (1)^{1/n} = (\cos 0° + i \sin 0°)^{1/n}$

$$= \cos\frac{2m\pi}{n} + i\sin\frac{2m\pi}{n}$$

$z = e^{i\frac{2m\pi}{n}}$ where m = 0, 1, 2 ... n – 1

Let $z_1 = 1$ and $z_2 = e^{i\frac{2m\pi}{n}}$

Whre $z_2 - 0 = (z_1 - 0)\, e^{i\frac{\pi}{2}}$

$$e^{i\frac{2k\pi}{n}} = 1 \,.\, e^{-i\frac{\pi}{2}} \Rightarrow n = 4k$$

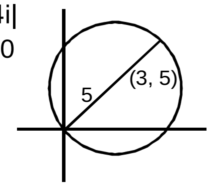
Sol.7 **(a)** Since $1 + \omega + \omega^2 = 0$, the given determinant

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = \begin{vmatrix} 3 & 0 & 0 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$

by change $R_1 \to R_1 + R_2 + R_3$

(b) $||z_2| - 5| \le |z_2 - 3 - 4i|$

$\Rightarrow ||z_2| - 5| \le 5 \Rightarrow |z_2| \le 10$



Aliter : $|z_2|$ maximum = 10

$|z_1 - z_2| \le |z_1| - |z_2|$

$\le 12 - 10$

$|z_1 - z_2| \le 2$

(c) $z^{p+q} - z^p - z^4 + 1 = 0$

$(z^p - 1)(z^q - 1) = 0$

Either α is a p^{th} root of unity of q^{th} roots of unity using the prop. of n^{th} root of unity

either $1 + \alpha + \alpha^2 + ... + \alpha^{p-1} = 0$

$1 + \alpha + \alpha^2 + ... + \alpha^{2-1} = 0$

If both the equation hold simultaneously, wiothout loss of generalisation let p > r

$\therefore\ 1 + \alpha + \alpha^2 + ... + \alpha^{p-1} = 0$

$\Rightarrow 1 + \alpha + \alpha^2 + ... + \alpha^{n-1} + \alpha^q + \alpha^{q+1} + ... + \alpha^{p-1} = 0$

$\Rightarrow 0 + \alpha^q + \alpha^{q+1} + ... + \alpha^{p-1} = 0$

Now, $\alpha^q = 1$

$\Rightarrow 1 + \alpha + \alpha^2 + ... + \alpha^{p-q-1} = 0$

Hence a should be the $(p - q)^{th}$ root of unity

i.e. $\alpha^{p-1} = 1$

$\Rightarrow$ p – q is a multiple of q ($\because$ q is prine)

i.e. p – q = n4

p = (n + 1) 4

$\Rightarrow$ p is not prine which is a controdiction H.P.

Sol.8 **(a)** $|z_1| < 1 < |z_2|$

T.P.T. $\left|\frac{1 - z_1\bar{z}_2}{z_1 - z_2}\right|$

$|1 - z_1\bar{z}_2| < |z_2 - z_1|$

$(1 - z_2\bar{z}_2)(1 - \bar{z}_1 z_2) < (z_2 - z_1)(\bar{z}_2 - \bar{z}_1)$

$1 + |z_1|^2 |z_2|^2 - |z_2|^2 - |z_1|^2 - |z_1|^2 < 0$

$(1 - |z_1|^2) + (|z_1|^2 - 1)|z_2|^2 < 0$

$(1 - |z_1|^2)(1 - |z_2|^2) < 0$

Which is true because $|z_1| < 1 < |z_2|$ H.P.

(b) $1 = |\Sigma a_i z^i| \le \Sigma |a_i z^i|$

$\Rightarrow 1 \le |a_1 z| + |a_2 z^2| + |a_3 z^3| + ... + |a_n z^n|$

$< 2(|z| + |z|^2 + |z|^3 + ... |z|^n)$

$$1 + |z| + |z|^2 + |z|^3 + ... + |z|^n > \frac{3}{2}$$

Case - I

$|z| < 1$

$\Rightarrow\ 1 + |z| + |z|^2 + \ldots \infty > \dfrac{3}{2}$

$\dfrac{1}{1-|z|} > \dfrac{3}{2} \qquad \Rightarrow\ |z| > \dfrac{1}{3}$

Case - II

$|z| \geq 1$

Obviously $|z| < \dfrac{1}{3}$ is not possible

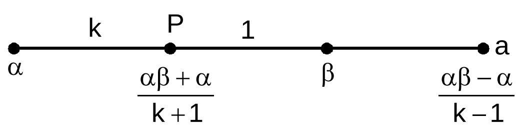
Hence $|z| < \dfrac{1}{3}$ and $\sum\limits_{i=1}^{n} a_i z^i = 1$ can not occur simultaneously for any a_i, $|a_i| < 2$

Sol.9 **(a)** $(1 + \omega^2)^m = (1 + \omega^4)^m$

$(1 + \omega^2)^m = (1 + \omega)^m$

$(-\omega)^m = (-\omega^2) \quad \Rightarrow\ m = 3$ possible

(b)



Centre is the mid-point of points duciding join of α and β is the ratio k : 1 intarnally & enternally

i.e. $\quad z = \dfrac{1}{2}\left[\dfrac{k\alpha+\alpha}{\alpha+1} + \dfrac{k\beta-\alpha}{k-1}\right] = \dfrac{\alpha - k^2\beta}{1-k^2}$

radius $= \left|\dfrac{\alpha-\alpha^2\beta}{1-k^2} - \dfrac{k\beta+\alpha}{1+ka}\right| = \left|\dfrac{k(\alpha-\beta)}{1-k^2}\right|$

Aliter : $\left|\dfrac{z-\alpha}{z-\beta}\right| = k$

$|z - \alpha|^2 = k^2\,|z - \beta|^2$

$(z-\alpha)(\bar{z} - \bar{\alpha}) = k^2\,(z-\beta)\,(\bar{z} - \bar{\beta})$

$z\bar{z} - \alpha\bar{z} - \bar{\alpha}z + \alpha\bar{\alpha} = k^2\,(z\bar{z} - \beta\bar{z} - \bar{\beta}z + \beta\bar{\beta})$

$z\bar{z} - \left(\dfrac{a-k^2\beta}{1+k^2}\right)\bar{z} - \left(\dfrac{\bar{\alpha}-k^2\bar{\beta}}{1-k^2}\right)z + \dfrac{\alpha\bar{\alpha}-k^2\beta\bar{\beta}}{1-k^2} = 0$

Which represent a circle with

centre $\dfrac{\alpha+k^2\beta}{1-k^2}$ and radius

$= \sqrt{\dfrac{(\alpha-k^2\beta)(\bar{\alpha}-k^2\bar{\beta})}{(1-k^2)^2} - \dfrac{\alpha\bar{\alpha}-k^2\beta\bar{\beta}}{(1-k^2)}} = \left|\dfrac{k(\alpha-\beta)}{1-k^2}\right|$

Sol.10 **(a)** The point (1, 0), $(\sqrt{2}-1, -\sqrt{2})$ and $(\sqrt{2}-1, \sqrt{2})$ are equidistant from the point (–1, 0)

The shaded area belongs to the region outside the sector of circle $|z + 1| = 2$, lying between the line rays

$\arg(z+1) = \dfrac{\pi}{4}$ and $\arg(z+1) = -\dfrac{\pi}{4}$

(b) $|a + b\omega + \cos^2| = \sqrt{\left(a-\dfrac{b}{2}-\dfrac{c}{2}\right)^2 + \dfrac{3}{4}(c-b)^2}$

$= \sqrt{\dfrac{1}{2}((a-b)^2 + (b-c)^2 + c(c-a)^2)}$

This is minimum when a = b and $(b-c)^2 = (b-c)^2 = (c-a)2 = 1 \quad \Rightarrow$ minimum value = 1

(c) Since centre of circle i.e. (1, 0) is also the mid point of diagonals of square

$\Rightarrow\ \dfrac{z_1+z_2}{2} = z_0$

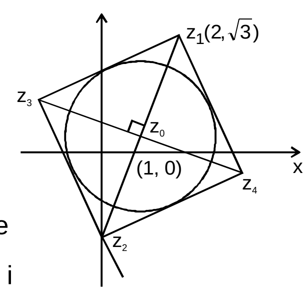
$\Rightarrow\ z_2 = -\sqrt{3}\,i$

and $\dfrac{z_3-1}{z_1-1} = e^{\pm\frac{i\pi}{2}}$

$\Rightarrow$ other vertices are

$z_3, z_4 = (1 - \sqrt{3}) + i$

and $(1 + \sqrt{3}) - i$



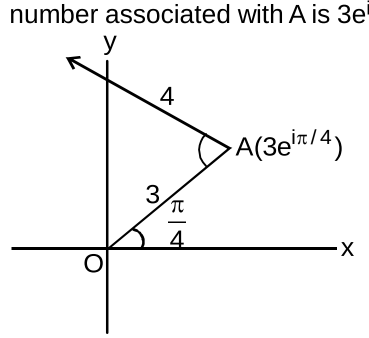
Sol.11 $\omega = \alpha + i\beta$

$\dfrac{\omega - \bar{\omega}z}{1-z}$ is purely real

$\Rightarrow\ \dfrac{\omega-\bar{\omega}z}{1-z} = \dfrac{\bar{\omega}-\omega\bar{z}}{1-\bar{z}} \qquad \Rightarrow (z\bar{z} - 1)\,(\bar{\omega} - \omega) = 0$

$\Rightarrow\ z\bar{z} = 1 \Rightarrow\ |z|^2 = 1 \Rightarrow\ |z| = 1$

Sol.12 **(a)** Let OA = 3, so that the complex number associated with A is $3e^{i\pi/4}$



If z is te complex number associated with P,

then $\dfrac{z-3e^{i\pi/4}}{-3e^{i\pi/4}} = \dfrac{4}{3}\ e^{-i\pi/2} = \dfrac{-4i}{3}$

$\Rightarrow 3z - 9e^{i\pi/4} = 12\ ie^{i\pi/4}$

$z = (3 + 4i)\ e^{i\pi/4}$

(b) $|z| = 1\ ;\ z \neq \pm 1$

Let $z = \cos\theta + i\sin\theta$

$z^2 = \cos^2\theta - \sin^2\theta + 2i\sin\theta\cos\theta$

$= \cos 2\theta + i\sin 2\theta$

$$\frac{z}{1-z^2} = \frac{\cos + i\sin\theta}{1-(\cos 2\theta + i\sin 2\theta)} = \frac{e^{i\theta}}{1-e^{2i\theta}}$$

$$= \frac{1}{e^{i\theta} - e^{i\theta}} = \frac{-1}{2\sin\theta} = \frac{i}{2\sin\theta}$$

z lies on imaging axis

Aliter : $E = \dfrac{z}{1-z^2} = \dfrac{z}{z\bar{z} - z^2} = \dfrac{1}{\bar{z} - z}$

Which is imaging

Sol.13 **(a)** $z_0 = (1 + 2i)$

$$\frac{x - x_1}{\cos 45^o} = \frac{y - y_1}{\sin 45^o} = \sqrt{2}$$

$x - x_1 = 1,\ \ y - y_1 = 1$

$x = 7,\ \ y = 6\ \ \ P(7, 6)$

Now Rotate P by an angle of $\dfrac{\pi}{2}$

$z_2 = i\ (7 + 6i) = -6 + 7i$

(b) A = Set of points on and above theline y = 1 is the argand plane

B = set of points on the circle $(x-2)^2 + (y-1)^2 = 9$

$C = \text{Re}((1-i)z) = \text{Re}(1-i)\ (x + iy) = \sqrt{2}$

$x + y = \sqrt{2}$

(i) Hence $(A \cap B \cap C)$ = has only one point of intersection

(ii) The points (–1, 1) and (5, 1) ar the extrmities of a diameter of the given circle

Hence $|z + 1| - i^2 + |z - 5 - i| = 36$

(iii) $||z| - |\omega_1|| < |z - \omega|$

and $|z - \omega|$ = Distance between z and ω

z is fixed. Hence distance between z and ω would be maximum for diametrically apposite point

$\Rightarrow\ |z - \omega| < 6 \quad \Rightarrow\ -6 < |z| - |\omega| < 6$

$-3 < |z| + \omega| + 3 < 9$

Sol.14 $\bar{z}z^3 + z\bar{z}^3 = 350$

$z\bar{z}\,(\bar{z}^2 + z^2) = 350$

Put $z = x + iy$

$(x^2 + y^2)\ (x^2 - y^2) = 175$

$(x^2 + y^2)\ (x^2 - y^2) = 5.5.7$

$x^2 + y^2 = 25$

$x^2 - y^2 = 7$

$x = \pm 4,\ y = \pm 3$

$x, y \in I$

Area = 8 × 6 = 48 sq. units

Sol.15 $z = \cos\theta + i\sin\theta$

$x = \sin\theta + \sin\theta + ... + \sin 29\theta$

$(2\sin\theta)\ x = 1 - \cos 2\theta + \cos 2\theta - \cos 2\theta + ... + \cos 28\theta - \cos 30\theta$

$$x = \frac{1 - \cos 30\theta}{2\sin\theta} = \frac{1}{4\sin 2^o}$$

Sol.16 **B**

$\alpha^3 + \beta^2 = q$

$\Rightarrow\ (\alpha + \beta)^3 - 3\alpha\beta\,(\alpha + \beta) = 4$

$\Rightarrow\ -p^3 + 3p\alpha\beta = q \Rightarrow \alpha\beta = \dfrac{q + p^3}{3p}$

$$x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta}\,.\,\frac{\beta}{\alpha} = 0$$

$$x^2 - \frac{(\alpha^2 + \beta^2)}{\alpha\beta}\ x + 1 = 0$$

$$x^2 - \left(\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}\right)x + 1 = 0$$

$$x^2 - \frac{p^2 - 2\left(\dfrac{p^3 + q}{}\right)}{3P}x + 1 = 0$$

$\Rightarrow (p^3 + q)\ x^2 - (3p^3 - 2p^3 - 2q)\ x + (p^3 + q) = 0$

$\Rightarrow (p^3 + q)\ x^2 - (p^3 - 2q)\ x\ (p^3 + q) = 0$

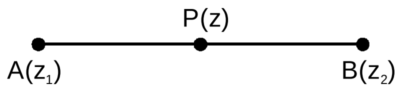
Sol.17 r_1, r_2, r_3 Î {1, 2, 3, 4, 5, 6}

r_1, r_2, r_3 are of the form 3k, 3k +1, 3k + 2

Reqd. probability $= \dfrac{3! \times {}^2C_1 \times {}^2C_1 \times {}^2C_1}{6 \times 6 \times 6}$

$$= \frac{6 \times 8}{216} = \frac{2}{9}$$

Sol.18 Given $z = (1 - t)\, z_1 + tz_2$



$$\Rightarrow \frac{z - z_1}{z_2 - z_1} = t \Rightarrow \arg\left(\frac{z - z_1}{z_2 - z_1}\right) = 0 \text{ ...(1)}$$

$$\Rightarrow \arg(z - z_1) = \arg(z_2 - z_1)$$

$$\frac{z - z_1}{z_2 - z_1} = \frac{\bar{z} - \bar{z}_1}{\bar{z}_2 - \bar{z}_1}$$

$$\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$$

AP + PB = AB $\Rightarrow |z - z_1| + |z - z_2| = |z_1 - z_2|$

Sol.19 $\omega = e^{i2\pi/3}$

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

$$z\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z+\omega^2 & 1 \\ 1 & 1 & z+\omega \end{vmatrix} = 0$$

$$z\,[(z + \omega^2)(z + \omega) - 1 - \omega(z + \omega + 1) + \omega^2(1 - 2 - \omega^2)] = 0$$

$\Rightarrow z^3 = 0$

z = 0 is only solution

Sol.20 **(a)** $\left|\frac{z}{|z|} - i\right| = \left|\frac{z}{|z|} + i\right| = z \neq 0$

$\left|\frac{z}{|z|}\right|$ is unimodular complex number

and lies on $\perp^n$ bisector of i and – i

$\Rightarrow \left|\frac{z}{|z|}\right| + \pm 1 \Rightarrow z = \pm |z| \Rightarrow$ z is real number

Img (z) = 0

(b) $|z + 4| + |z - 4| = 10$

z lie son an ellipse whoe focus are (4, 0) and (–4, 0) and length of major axis is 10.

$\Rightarrow 2ae = 8 = 0$ and $2a = 10 \qquad \Rightarrow e = \frac{4}{5}$

$|\text{Re}(z)| \leq 5$

(c) $|\omega| = 2 \Rightarrow \omega = 2(\cos\theta + i\sin\theta)$

$$x + iy = 2(\cos\theta + i\sin\theta) - \frac{1}{2}(\cos q - i\sin\theta)$$

$$= \frac{3}{2}\cos\theta + i\,\frac{5}{2}\sin\theta$$

$$\Rightarrow \frac{x^2}{\left(\frac{3}{2}\right)^2} + \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$$

$$e^2 = 1 - \frac{\frac{3}{4}}{\frac{25}{4}} = 1 - \frac{9}{25} \quad \Rightarrow e = \frac{4}{5}$$

(d) $|\omega| = 1 \Rightarrow x + iy = \cos\theta + i\sin\theta + \cos\theta - i\sin\theta$

$x + iy = 2\cos\theta$

$|\text{Re}(z)| \leq 1 \qquad \text{Im}(z) = 0$

Sol.21 **D**

Given

$a + 8b + 7c = 0$ .....(i)

$9a + 2b + 3c = 0$ .....(ii)

$7(a + b + c) = 0$ .....(iii)

On solving $a = -k/7$, $b = -6k/7$, $c = k$

Put in the given plane $k = -7$

So $7a + b + c = 6$

Sol.22 **A**

$a = 2 \Rightarrow -k/7 = 2 \qquad \Rightarrow k = -14$

$b = +12$

$c = -14$

Put in $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c} = -2$

Sol.23 **B**

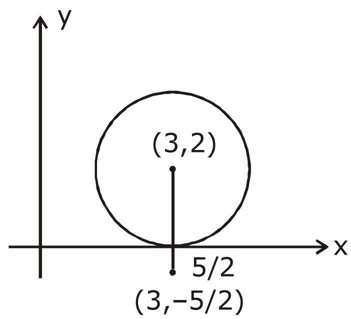
$b = 6 \Rightarrow K = -7$

$a = 1, c = -7$

$$S = \sum_{n=0}^{\infty}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n = \sum_{n=0}^{\infty}\frac{(\alpha+\beta)^n}{\alpha^n\beta^n}$$

$$= \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n = 7 \qquad \begin{cases} \alpha + \beta = -b/a \\ \alpha . \beta = c/a \end{cases}$$

Sol.24



$$2\,|z-(3-5/2\,i)| = 0005$$

Sol.25 A

Value of the determinant

$= 1 - \omega(a + c) - ac\omega^2$

total matrices = 2

Sol.26. $\dfrac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ on putting the value of x, y, z directly everything is getting cancelled out

Sol.27

(A) $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

but the interior angle must be

$\pi - \pi/3 = 2\pi/3$

(B) $\displaystyle\int_a^b (f(x) - 3x)\,dx = a^2 - b^2$

$$\Rightarrow \int_a^b f(x)dx - \left.\frac{3x^2}{2}\right|_a^b = a^2 - b^2$$

$$\Rightarrow \int_a^b f(x)dx = \frac{3(b^2-a^2)}{2} - (b^2 - a^2) = \frac{(b^2-a^2)}{2}$$

$F(x) = F(b) - F(a) \Rightarrow F(x) = \dfrac{x^2}{2}$

differentiating w.r.t x we get f(x) = x

$$\Rightarrow f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$$

(C) Let $I = \dfrac{\pi^2}{\ln 3}\displaystyle\int_{7/6}^{5/6} \sec(\pi x)\,dx$

put $\pi x = t \Rightarrow \pi\,dx = dt$

$$I = \frac{\pi}{\ln 3}\int_{7\pi/6}^{5\pi/6} \sec t\,dt$$

$$I = \frac{\pi}{\ln 3}\left\{\ln(\sec t + \tan t)\right\}_{7\pi/6}^{5\pi/6}$$

$I = \pi$

(D) $z = e^{i\theta}$

$$\Rightarrow \left|\text{Arg}\,\frac{1}{1-2}\right| = \left|\text{Arg}\,\frac{1}{1-e^{i\theta}}\right|$$

$$= \left|\text{Arg}\left(\frac{1}{2} + \frac{1}{2}\,i\cot\frac{\theta}{2}\right)\right|$$

maximum value of $\theta = \pi/2$

Sol28.

(A) $z = e^{i\theta}$

$$\Rightarrow \text{Re}\left(\frac{2iz}{1-z^2}\right) = \frac{1}{\sin\theta}$$

(B) $f(x) = \sin^{-1}\left(\dfrac{8.3^x}{9-(3^x)^2}\right)$

$-1 \le \dfrac{8.3^x}{9-(3^x)^2} \le 1$ Let $3^x = t$

$-1 \le \dfrac{3.t}{9-t^2} \le 1$

on solving $x \in (-\infty, 0] \cup [2, \infty)$

(C) $f(\theta) = 2\sec^2\theta$

(D) $f'(x) = \dfrac{3}{2}x^{1/2}(5x - 10) \ge 0,\ \forall\, x \ge 2$

$x \in [2, \infty)$

Sol.29 D

$\text{Im}(z) \ne 0$ $a = z^2 + z + 1$

$z = \alpha + i\beta$ $\beta \ne 0$

$a = \alpha^2 - \beta^2 + 2i\alpha\beta + \alpha + i\beta + 1$

$= \alpha^2 - \beta^2 + \alpha + 1 + i\beta(2\alpha + 1)$

$\alpha = -1/2 = 3/4 - \beta^2$